truss structure. This structure has been used for many structural optimization methodology demonstrations including one utilizing neural networks.[8] The nominal design for the structure without active members typically consists of all 10 aluminum members having a cross-sectional area of 1.0 $in.^2$ Active members were substituted for element number 1 (the bottom root longeron) and for element number 8 (the upwardly pointing root diagonal). This baseline design with active members has natural frequencies of 13.6, 39.0, 40.2, 75.6, 82.3, 93.0, and 94.0 Hz.

Transfer functions between the active member actuators and sensors were generated. A typical set of transfer functions for the two active members is shown in Fig. 3. The location of the poles and zeroes relative to their location for the undamaged structure gives an indication of the health of the remainder of the structure.

Input training data for the neural network consisted of the imaginary parts of the transfer function poles and zeroes. Output data from the neural network consisted of the cross-sectional areas of each of the 10 bars in the truss. A complete set of training data was obtained by sequentially decreasing the stiffness of each member of the truss by a known amount and presenting the resulting input and output training data just described to the neural network.

All results presented below were obtained using a neural network with a single-hidden layer of 14 tangent sigmoidal neurons. Additional configurations of neural networks were trained and used to locate and predict the damage in the 10-bar truss, but did not achieve better results than the single-layer, 14-neuron network. Two networks that achieved approximately equivalent results were a double-hidden layer network (with 5 and 4 tangent sigmoidal neurons) and a single-hidden layer network with 17 log sigmoidal neurons.

Table 1 contains a list of the simulated damage cases that were run on the 10-bar truss structure. The resulting neural network predictions of the member cross-sectional areas are also given in Table 1. Test case 1 represents a condition where a single member was damaged (i.e., member number 4). This type of damage is within the domain of the training data and gives an indication of the adequateness of the training of the neural network. The damage assessment from the neural network indicates that member 4 is damaged and the predicted level of damage, $A_4 = 0.66$, compares well with the actual level of damage used to generate the damaged structure transfer functions. The network also predicts slight damage to members 2 and 9, which is a result of the static indeterminancy in the 10-bar truss. Test cases 2 and 3 represent multiple member damage conditions where 2 and 3 members are damaged simultaneously, respectively. These types of damage are outside the domain of the training data of the neural network. Nonetheless, the neural network pinpoints the damage very well for both cases. Furthermore, the level of damage is predicted within a few percent for test case 2 and within approximately 10% for test case 3.

Conclusions

A methodology for detecting damage in structural systems has been described. The method utilizes the active members that are already present for a controlled structure in conjunction with a trained artificial neural network. A numerical example demonstrated the feasibility of the method which pinpoints the damaged members and which gives a very good estimate of the damage present in each member. The keys to making the problem tractable for larger problems are adequately identifying the members at high risk for potential damage and including enough pole/zero information in the training of the neural network.

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Optimal Fail-Safe Design of Elastoplastic Structures

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Introduction

THE objective of this work is to develop a consistent and broadly applicable model for elastoplastic fail-safe structural optimization. Toward this end, the multicriteria model for the optimal fail-safe design of structures presented in Ref. 1 (see also Ref. 2) is extended to treating elastoplastic design problems based on a new general elastoplastic analysis variational formulation as applied to two-dimensional truss structures (see Taylor[3]).

The new formulation is capable of covering the whole loading range of a structure, i.e., from zero load to limit load. The basis for the formulation is the assertion that the load factor for a proportional loading case assumes a maximum relative to all statically admissible stress states for a given bound on the complementary energy (i.e., it corresponds to the lower bound theorem of limit load analysis). The model for this formulation, expressed here for a two-dimensional truss composed of M bars, is given in terms of the vector $t \in R^M$ of normalized (with respect to the yield stress σ_Y) bar stresses, and the load factor λ as follows:

$$\min_{t_k} -\lambda \tag{1}$$

subject to

$$\begin{cases} \sum_{j=1}^{M} C_{ji} t_j - \lambda f_i = 0 & \\ t_k - 1 \le 0 & i = 1, \ldots, 2I \\ -t_k - 1 \le 0 & k = 1, \ldots, M \\ \frac{\sigma_Y^2}{2} \sum_{n=1}^{M} \sum_{m=1}^{M} t_n D_{nm} t_m - \varepsilon^2 \le 0 & \end{cases}$$

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where $C \in R^{2I\times M}$ is the force equilibrium matrix that depends on the nodal coordinates of the I free degrees of freedom of the structure and on the cross-sectional areas of the bars, f_i is the external force component acting along the direction of the ith degree of freedom. $D \in R^{M\times M}$ is a diagonal design matrix such that $D_{kk} = A_k L_k / E$ with A_k the cross-sectional area of the kth element, is the Young's modulus for the structure's material. L_k is the length of the kth bar, and ε is the total complementary energy in the loaded structure.

The formulation just described is very powerful. Indeed, by treating ε as a parameter, problem statement (1) can be used to analyze any structure for all load intensities (keeping the load configuration fixed) within the range from zero load ($\lambda = 0$) through to collapse. Thus, by specifying an appropriate complementary energy level ε, it is possible to analyze the most general structures operating within the elastoplastic range. It should be noted that the material linearly elastic–perfectly plastic constitutive relation is implicit within the specification of the set of bounds stated in problem (1).

Problem Formulation

The concept of fail-safe design implies that a structure is required and supposed to perform satisfactorily under various sets of contexts (circumstances) in addition to the primary purpose for which it is to be designed. It is therefore clear that the general fail-safe design problem is a multicriteria design problem with possibly more than one criterion corresponding to each context.

Based on Eq. (1), the elastoplastic fail-safe design problem formulation can be obtained assuming the various loading contexts are distinguished one from another by the loading configuration as well as by the desired maximum complementary energy level prescribed for the structure via

$$\max_{A_k}\left[\min_{\alpha,\, t_{\alpha_k}}\{-a_\alpha\lambda_\alpha\}\right] \tag{2}$$

subject to:

$$\begin{cases} \displaystyle\sum_{j=1}^{M} C_{\alpha_{ji}} t_{\alpha_j} - \lambda_\alpha f_{\alpha_i} = 0 & \\ t_{\alpha_k} - 1 \le 0 & \\ -t_{\alpha_k} - 1 \le 0 & \alpha = 1,\dots,P \\ \displaystyle\sum_{k=1}^{M} A_k L_k - \boldsymbol{R} \le 0 & i = 1,\dots,2I \\ \underline{A}_k - A_k \le 0 & k = 1,\dots,M \\ \displaystyle\frac{\sigma_Y^2}{2}\sum_{n=1}^{M}\sum_{m=1}^{M} t_{\alpha_n} D_{\alpha_{nm}} t_{\alpha_m} - \varepsilon_\alpha^2 \le 0 & \end{cases}$$

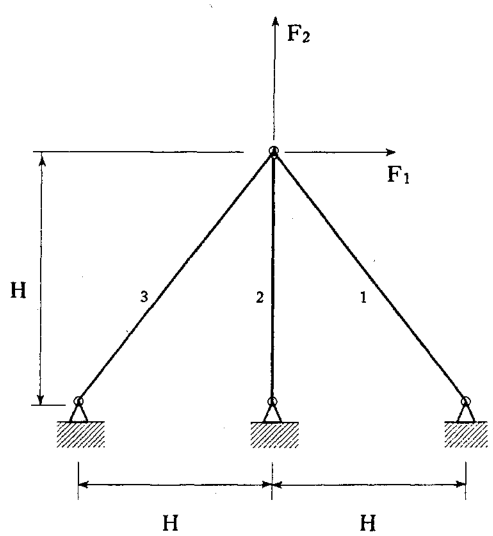

Fig. 1 Three-bar truss for the computational example.

where α denotes the context number, $\underline{A}_k$ is the lower bound on the design of the kth bar, and $\boldsymbol{R}$ is the available resource. Statement (2) represents the maximization of the vector of a structure's weighted load carrying capacities under the action of P contexts with respect to design. The relative values of the weights a_α represent the relative importance of each context and correspond to a point in the Edgeworth-Pareto set (see Ref. 4).

The scalar version of the multicriteria design problem of Eq. (2) may be obtained via the implementation of the bound method (e.g., Ref. 2) to yield

$$\min_{A_k} -\beta \tag{3}$$

subject to

$$\begin{cases} \beta - a_\alpha\lambda_\alpha \le 0 & \\ \displaystyle\sum_{j=1}^{M} C_{\alpha_{ji}} t_{\alpha_j} - \lambda_\alpha f_{\alpha_i} = 0 & \\ t_{\alpha_k} - 1 \le 0 & \alpha = 1,\dots,P \\ -t_{\alpha_k} - 1 \le 0 & i = 1,\dots,2I \\ \displaystyle\sum_{k=1}^{M} A_k L_k - \boldsymbol{R} \le 0 & k = 1,\dots,M \\ \underline{A}_k - A_k \le 0 & \\ \displaystyle\sum_{n=1}^{M}\sum_{m=1}^{M} t_{\alpha_n} D_{\alpha_{nm}} t_{\alpha_m} - \hat{\varepsilon}_\alpha^2 \le 0 & \end{cases}$$

with $\hat{\varepsilon}_\alpha = \sqrt{2}\varepsilon_\alpha/\sigma_Y$. Specifying the value for the allowed complementary energy level ε_r for the rth context determines the regime of operation for the structure under the corresponding context. This means that for contexts for which the specified ε_α is smaller than the complementary energy corresponding to yield (ε_Y) the structure's maximum load carrying capacity is sought in the elastic range while for those with ε_α greater than ε_Y the response is expected to be in the elastoplastic regime. For cases that ε_α is greater than the complementary energy corresponding to global failure (ε_L) the load carrying capacity is associated with the ultimate load. Nevertheless, due to the fact that the design of the structure is not known a priori, the complementary energy for each context should be treated as a parameter in the problem.

The character of the solutions to problem (3) can be deduced from the corresponding necessary conditions for optimality. The Lagrangian for the problem is given by

$$\begin{aligned} L = -\beta + \sum_{\alpha=1}^{P}\Bigg\{ & \Gamma_\alpha(\beta - a_\alpha\lambda_\alpha) + \sum_{i=1}^{2I}\sum_{j=1}^{M} \nu_{\alpha_i}(C_{\alpha_{ij}} t_{\alpha_j} - \lambda_\alpha f_{\alpha_i}) \\ & + \sum_{k=1}^{M}\mu^{+}_{\alpha_k}(t_{\alpha_k} - 1) + \sum_{k=1}^{M}\mu^{-}_{\alpha_k}(-t_{\alpha_k} - 1) \\ & + \Lambda_\alpha\left(\sum_{n=1}^{M}\sum_{m=1}^{M} t_{\alpha_n} D_{nm} t_{\alpha_m} - \hat{\varepsilon}_a^2\right)\Bigg\} + \sum_{k=1}^{M}\eta_k(\underline{A}_k - A_k) \\ & + \Omega\left(\sum_{k=1}^{M} A_k L_k - \boldsymbol{R}\right) \end{aligned} \tag{4}$$

where Γ_α, ν_{α_1}, $\mu^{+}_{\alpha_k}$, $\mu^{-}_{\alpha_k}$, Λ_α, η_k, and Ω are Lagrangian multipliers. The Kurash-Kuhn-Tucker necessary conditions for optimality correspond to variations of the Lagrangian (4) with respect to β,

λ_α, $\nu_{\alpha 1}$, t_k, and A_k, repectively, together with the following complementarity conditions:

$$\Gamma_\alpha(-a_\alpha\lambda_\alpha-\beta) = 0, \quad \Gamma_\alpha \ge 0 \quad \text{and} \quad -a_\alpha\lambda_\alpha-\beta\le 0 \tag{5a}$$

$$\mu^+_{\alpha_k}(t_{\alpha_k}-1) = 0, \quad \mu^+_{\alpha_k}\ge 0 \quad \text{and} \quad t_{\alpha_k}-1\le 0 \tag{5b}$$

$$\mu^-_{\alpha_k}(-t_{\alpha_k}-1) = 0, \quad \mu^-_{\alpha_k}\ge 0 \quad \text{and} \quad -t_{\alpha_k}-1\le 0 \tag{5c}$$

$$\eta_k(\underline{A}_k-A_k) = 0, \quad \eta_k\ge 0 \quad \text{and} \quad \underline{A}_k-A_k\le 0 \tag{5d}$$

$$\Omega\left(\sum_{k=1}^{M}A_kL_k-\boldsymbol{R}\right) = 0, \quad \Omega\ge 0 \quad \text{and} \quad \sum_{k=1}^{M}A_kL_k-\boldsymbol{R}\le 0 \tag{5e}$$

$$\Lambda_\alpha\left(\sum_{n=1}^{M}\sum_{m=1}^{M}t_{\alpha_n}D_{nm}t_{\alpha_m}-\hat{\varepsilon}_a^2\right) = 0, \quad \Lambda_\alpha\ge 0 \quad \text{and}$$

$$\sum_{n=1}^{M}\sum_{m=1}^{M}t_{\alpha_n}D_{nm}t_{\alpha_m}-\hat{\varepsilon}_a^2\le 0 \tag{5f}$$

for $\alpha = 1, \ldots, P$ and $k = 1, \ldots, M$.

As can be established from the conditions (5a–f), for this type of problem the dual load vectors $\mu^+_{\alpha_k}$ or $\mu^-_{\alpha_k}$ in general are not mutually orthogonal. This fact is obvious due to the possibility that a member may be in a yielded state under more than one context. Moreover, $\mu^+_{\alpha_k}$ and $\mu^-_{\alpha_k}$ are orthogonal and, therefore, the structure's domain is composed of a subdomain for which the design variables assume their lower bound values $\underline{A}_k$ and of design subdomains where $A_k > \underline{A}_k$. Nevertheless, due to the global character of the criteria used in the problem, it is impossible to characterize the intervals constituting these subdomains unless the structure is in its elastic range for all of the contexts (in which case the subdomains are distinct). Therefore, in order to obtain the solution to Eq. (3), the utilization of a general computational mathematical programing code is required.

A consequence of the multicriteria formulation underlying problem statement (3) is the issue of context dominancy on the solution. A context is said to be dominant when its corresponding β constraint in Eq. (3) is tight. According to the first condition in Eq. (5), the solution may be dominated by either one context, some of the contexts, or all of the contexts depending on the choice of the weights a_α.

Computational Example

The truss structure used in the example is given in Fig. 1. The forces F_1 and F_2 correspond to the primary and the secondary contexts, respectively. The requirement in the primary context is for a

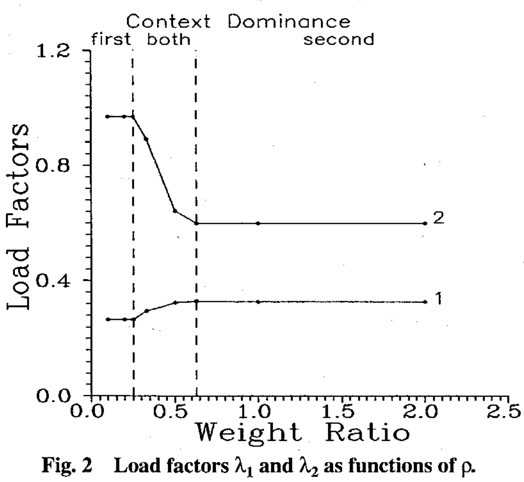

Fig. 2 Load factors λ_1 and λ_2 as functions of ρ.

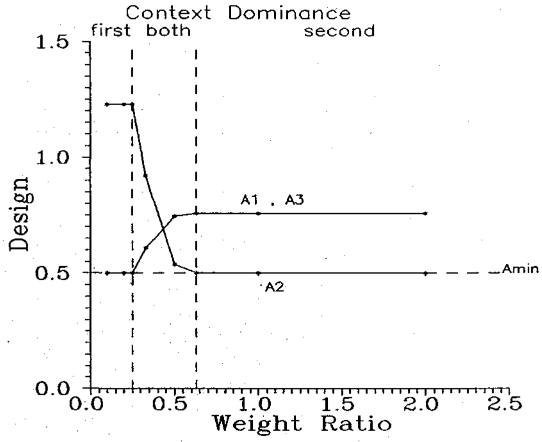

Fig. 3 Cross-sectional areas of the optimal bars as functions of ρ.

design that will yield as large as possible load carrying capacity under the action of F_1 while the structure remains elastic. The requirement in the secondary context is to find as large as possible collapse load with respect to force F_2. The former goal is utilized by assigning a low complementary energy level, while the latter is implemented by discarding the energy constraint corresponding to the secondary context. Figure 2 depicts the variation of the load factors λ_1 and λ_2 corresponding to each context as they depend on the weight ratio $\rho = a_2/a_1$. The corresponding cross-sectional areas of the bars are given in Fig. 3. The solutions plotted in Figs. 2 and 3 were obtained via the ADS subroutine (see Ref. 5).

The results indicate that ρ renders the design to be dominated by either the primary context alone (characterized by the fact that the cross-sectional area of member 2 assumes the value of the lower bound on design—consistent with the fact that this bar carries zero stress under the force F_1), or dominated by the secondary context alone (identified by the simultaneous yield of all three bars), or dominated by both contexts (yielding designs which consist of a compromise between the two designs just described).

Conclusions

The formulation of the problems of optimal fail-safe design of elastoplastic structures is unified by identifying the general fail-safe design problem as a multicriteria optimization problem of mathematical programming. The formulation presented in this work is based on a new general variational formulation of analysis as applied to two-dimensional truss structures.

The theory presented is general enough to accommodate structures that are composed of elements that have final flexural stiffness in addition to their axial stiffness. Such an extension will enable the design of fail-safe truss structures to take into account the possibility of local geometric instabilities as well as plastic yield.

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